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C. Rehbach

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NUMERICAL INVESTIGATION OF VORTEX SHEETS ISSUING FROM A SEPARATION LINE NEAR THE LEADING EDGE*

Colmar Rehbach **

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SUMMARY: Observation shows that, even for small incidences, the flow around slender delta wings separates along a line near the leading edge, forming a vortex sheet which rolls up into a pair of spiral vortices.

The theoretical treatment of this problem is possible for this simple form of wing, and has been carried out within the limits of such approximations as slender body theory and/or conical flow. Yet the phenomenon of building up of vortex sheets on the leading edge of lifting surfaces is encountered for more general geometric configurations for which an analytical treatment is out of the question.

For these configurations, it is proposed to use an iterative calculation method based on the substitution of the sheet of vorticity representing the wing and its trailing sheet by a network of concentrated line vortices. Though the proposed method might be used for wings of arbitrary shape, the results presented are limited to those of plane delta wings. They are compared with results obtained by purely analytical methods, and with experiments performed in the O.N.E.R.A. water-tunnel.

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** Research Engineer at the O.N.E.R.A.

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I. INTRODUCTION

For slender delta wings, one observes a leading edge separation starting at relatively small incidence angles which result in the formation of a vortex sheet which rolls up to form spirals. This flow was shown to exist by M. Roy [1] and led to many theoretical and experimental investigations. A detailed bibliography is given in a recent article by R. Legendre [2]. /326

Among the theoretical studies, the most elaborate one is the work of J. H. B. Smith [3] which in part is based on a previous theoretical study of K. W. Mangler and J. H. B. Smith [4] (an equivalent formulation without any numerical applications had already been given by R. Legendre [15]). Recently this analysis was extended to cambered delta wings (D. I. Pullin [5]) and to very slightly inclined delta wings (J. E. Barsby [6]). The modeling of the flow which allows the theoretical treatment is described in the work of Mangler and Smith. It is adapted in most of the investigations of leading edge vortices. It also constitutes the basis of the work described in the present article. Thus the flow is considered to be incompressible and inviscid. The wing is reduced to its skeleton and the separation line is assumed to be identical with the leading edge. Also there is no secondary separation.

With these hypotheses, the analytical study of the spiral sheets can be carried out within the framework of certain approximations, such as the approximation of slender bodies and the approximation of conical flow. The study of Mangler and Smith [4] is based on these two simplifying assumptions.

A. H. Sacks, et al. [7] calculated shapes of the sheets while only limiting themselves to the first hypothesis. It can be noted that in this case, the best results are obtained by using experimentally measured values for the vortex rate (shedding rate).

As R. Legendre has emphasized in his article [2] the phenomenon of sheet formation at the leading edge of lifting surfaces can be observed in much more general geometric configurations, for which the slender body and conical flow criteria are far from being satisfied. In order to make these configurations accessible to theoretical treatment, it is necessary to find new calculation methods.

For several years there has been a number of numerical methods [8, 9, 10] available which can be used in conjunction with large computers to calculate the equilibrium position of the vortex sheets which originate at the trailing edge and at the extremities of thin wings (lifting surfaces) having arbitrary shapes. These methods are characterized by dividing up the vortex surface using a network of concentrated vortices, which gives excellent results for this type of problem. The purpose of this article is to extend these methods to the particular case of separation at the leading edge.

II. SUMMARY OF THE FUNDAMENTALS OF THE NUMERICAL METHOD

We would like to recall that the aerodynamic effect of an infinitesimally thin lifting surface and of its sheet can be represented by a vortex layer. For the numerical calculation, this continuous analytical representation is replaced by a system of discrete vortices, in our case a system of horseshoe vortices. The geometry of the vortex system can be imposed on the wing. On the other hand, beyond the points where the vortex threads detach from the wing, they must be able to evolve freely so as to take on their equilibrium position defined by the condition of continuous pressure through the sheet.

The unknowns of the numerical scheme are the vortex intensities and the geometry of the sheet. They are interdependent and the problem formulated is nonlinear. The solution can be obtained by an iteration procedure.

One of the oldest methods is that of S. M. Belotserkovskii [8]. A faster method was proposed by D. J. Butter and G. J. Hancock [9], but it is limited in the original version to wings without edge vortices.

Based on these two methods, a calculation program was built at the O.N.E.R.A. It was designed to analyze thin wings of arbitrary shape but which only resulted in the formation of sheets from the trailing edge and from the extremities. It has been used for several years. We took into account some results given in two publications which appeared in Recherche Aérospatiale [11] (Aerospace Research). On a closer examination of these results on plane rectangular wings which are very slender, it can be seen that there is an edge zone which has a very pronounced conical character. This observation led us to develop the method presented here for the calculation of leading edge sheets.

III LEADING EDGE ITERATION METHOD OF CALCULATING A SHEET EMANATING FROM THE LEADING EDGE

The iteration procedure which we will describe in detail is in principle applicable to a large number of wing shapes. Our first results presented here were obtained during the development of the method and only involved plane wings having a slightly open triangular shape (slender delta wings). With this we were able to make comparisons with theoretical and experimental results given in other publications. This particularly simple geometric shape was used in most publications dealing with the separation close to the leading edge. On the other hand, this wing shape is well adapted to the analytical treatment and represents a severe test for the numerical method, because of the difficulties involved in calculating the velocity field using a very close grid of discrete singularities near the apex, which represents a singular point of the flow.

The success of our results obtained for the slender rectangular wing using the vortex scheme shown in Figure 1 led us to develop a similar scheme for the delta wing (Figure 2). One is also tempted to use the vortex distribution of linear theory for the first approximation, just like what is done for the rectangular wing. This results in an infinite value of ΔKp at the leading edge because of the twisting of the leading edge by the fluid. However, the variation of ΔKp, which is completely correct for the rectangular wing for which pressure continuity only has to be guaranteed along the trailing edge and the extremities, is very far removed from the desired solution for the delta wing with the formation of spiral vortices. In effect, for the delta wing, the condition $\Delta Kp = 0$ must be guaranteed for the entire contour. Experience shows that calculations which start with the linear solution do not converge to the solution which one wants to obtain here. In contrast to this, one hardly ever moves far away from a vortex thread solution which is not separated from the leading edge. The main problem consists of finding the first plausible approximation for the vortex distribution over the wing without making any hypotheses regarding the nature of the flow. We proceeded as follows:

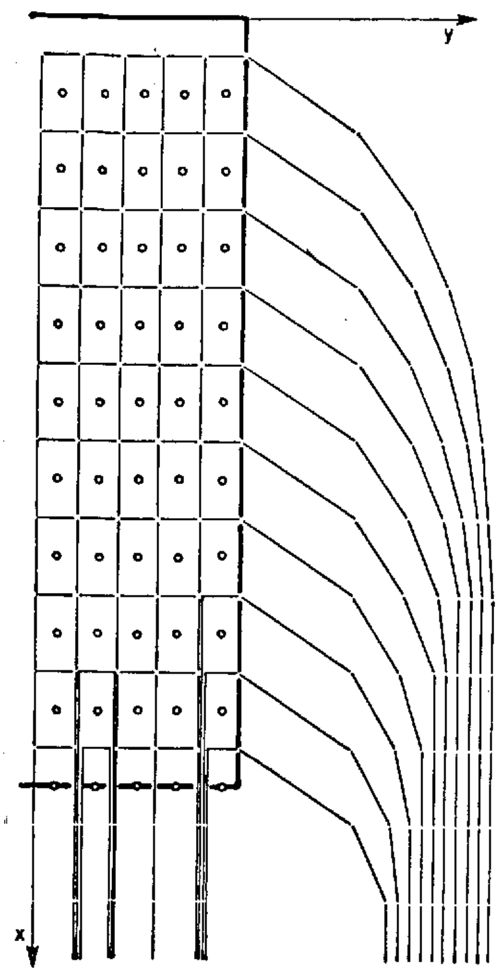

Figure 1. Distribution of horseshoe vortices along a rectangular wing

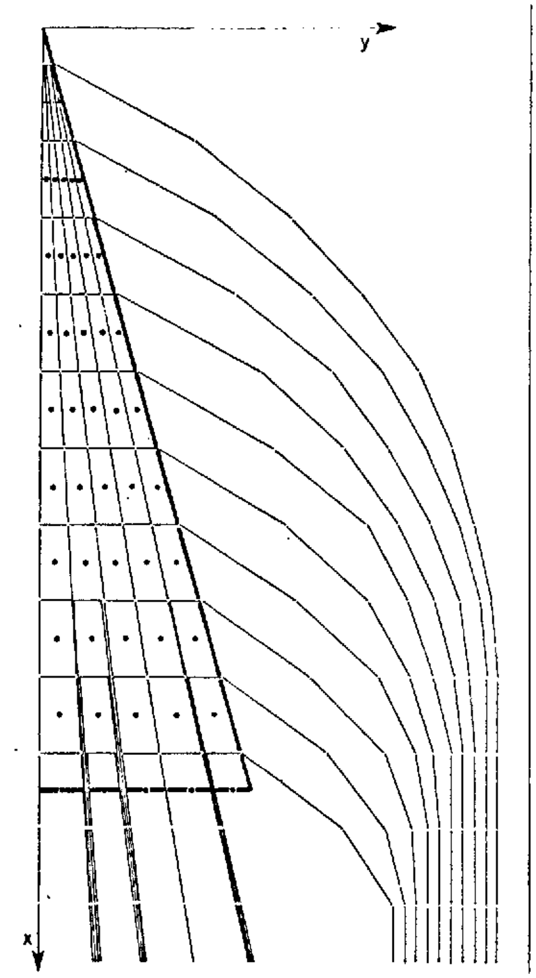

Figure 2. Distribution of horseshoe vortices over a delta wing

Starting with a geometric configuration having a twist at the leading edge and for which we guarantee the pressure continuity condition along the trailing edge and the extremities, we successively transfer to the configuration in which we are interested by reducing the part of the leading edge which has a twist. This is replaced by a part along which the pressure continuity condition is guaranteed. Figure 3 shows the procedure for the case of the delta wing. Here it is sufficient to /328

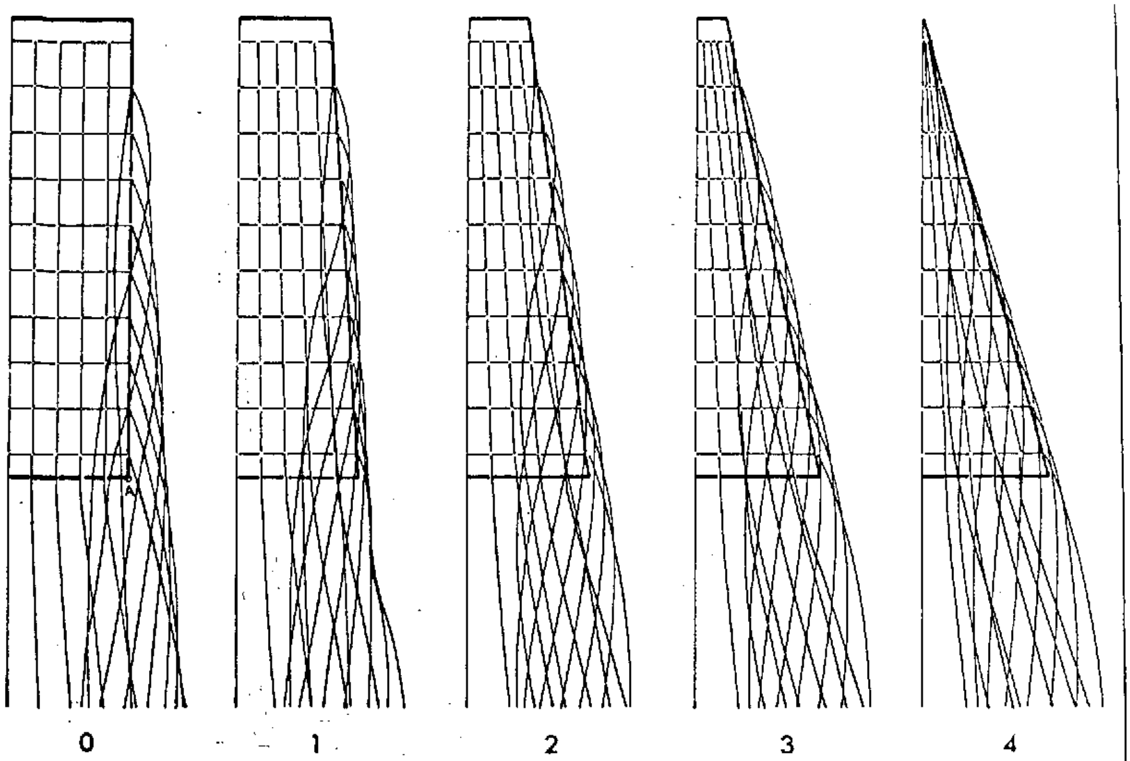

Figure 3. Calculation of a delta wing with leading edge separation by progressive deformation of a rectangular wing

progressively turn the line representing the wing extremity around the point A. For each intermediate geometric configuration, we evaluate the equilibrium position of the sheet and the associated vortex distribution using the procedure described above [11]. The initial data for the following sheet are the vortex distribution which is obtained by this method, as well as a sheet geometry derived from the preceding step by multiplying the transverse geometric values (y) by the ratio of the local span of the new and old configurations. The geometric values in the direction of the two other coordinate axes (x,z) are not changed. It is not necessary to have the same accuracy for the intermediate steps as for the definite configuration; the total number of iterations to be carried out remains acceptable.

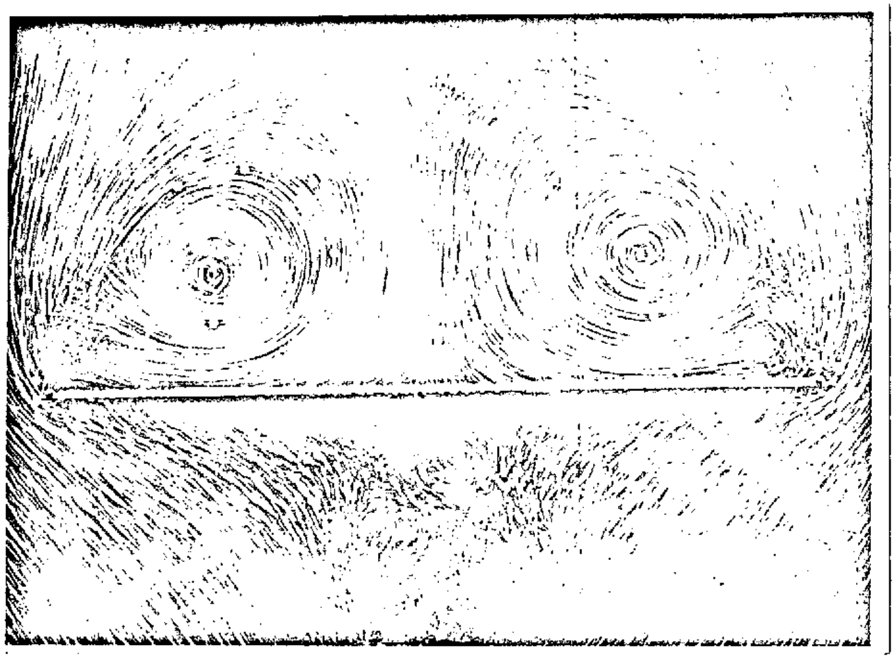

Figure 4. Trace of the sheet in the plane transverse to the trailing edge, compared with observation

IV. CALCULATION EXAMPLE

The example analyzed is a delta wing with a semi-aperture angle of $\varepsilon = 15°$. We will study the incidence angles $i = 7.5°$, $15°$ and $22.5°$ corresponding to the parameter $a = i/\varepsilon = 0.5$, 1 and 1.5 (Mangler and Smith used $\bar{a} = i/\mathrm{tg}\varepsilon$).

In Figure 4 we have superimposed our calculation results on an experimental photograph of H. Werlé, obtained in the hydrodynamic tunnel of the O.N.E.R.A. In the photograph, the trajectories of the flow in a plane perpendicular to the wing plane and located at the trailing edge are visualized by means of air bubbles suspended in water. The planform of the wing is identical to the wing calculated. The incidence angle during the experiment was $15°$ ($a = 1$) and the Reynolds number (reference length: central chord) was $2 \cdot 10^4$. The numerical results correspond to the intersection points with the same transverse plane of 20 vortex threads emanating from the leading edge. It was not possible to identify the air bubbles emanating from the leading edge during the experiments and therefore we must be content with a purely qualitative comparison.

Figure 5 shows the traces of the sheet for the incidence angles. $i = 7{,}5^{\circ}$ $(a = 0{,}5)$, 15° $(a = 1)$ and $22{,}5^{\circ}$ $(a = 1{,}5)$ in the plane perpendicular to the trailing edge.

In order to verify the degree to which the flow is conical, in Figure 6 we compare the sheet traces for the case **a** = 1 in three different planes (x/c = 0.50-0.75 and 1). If the flow were conical, these three curves would have to lie on top of one another. It can be seen that the deviation from conical flow is moderate.

A comparison of the shape of the sheet obtained using the O.N.E.R.A. program with those calculated using two analytical procedures is given in Figure 7. The result of J. H. B. Smith [3] corresponds to an infinite delta wing and therefore a conical flow which is calculated using the slender body theory. It only depends on the incidence angle i and the aperture angle ε through the ratio i/ε. The procedure of A. H. Sacks [7] also remains within the framework of slender body theory but does not use the hypothesis of conical flow conditions. His results, therefore, depend both on the incidence angle and on the apex semi-aperture angle (14°, aspect ratio 1). The deviation between the three curves is considerable. The position of the vortex core of the real flow obtained from the O.N.E.R.A. hydrodynamic tunnel is also indicated in the figure and corresponds to a sheet which is considerably above those of Smith and Sacks, and agrees better with our calculation. One explanation for the poor

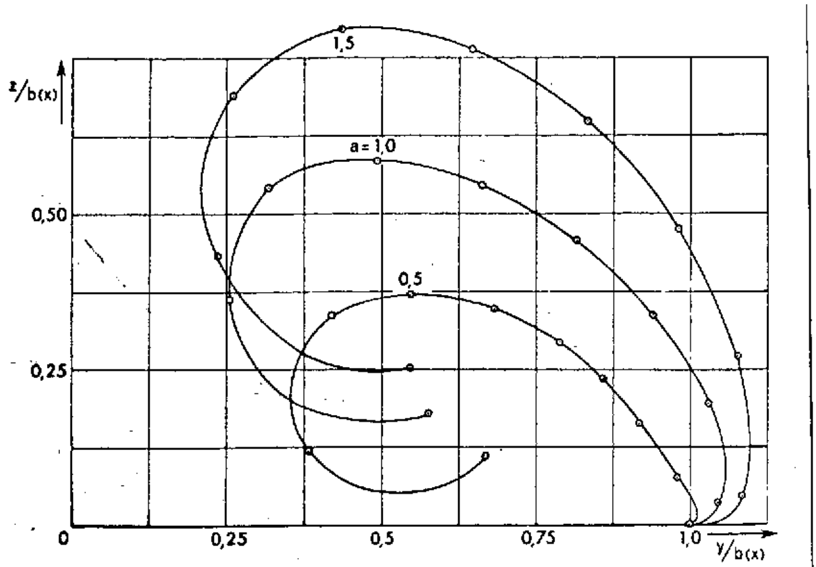

Figure 5. Evolution of the sheet in the plane transverse to the trailing edge for a delta wing with a 30° apex aperture angle

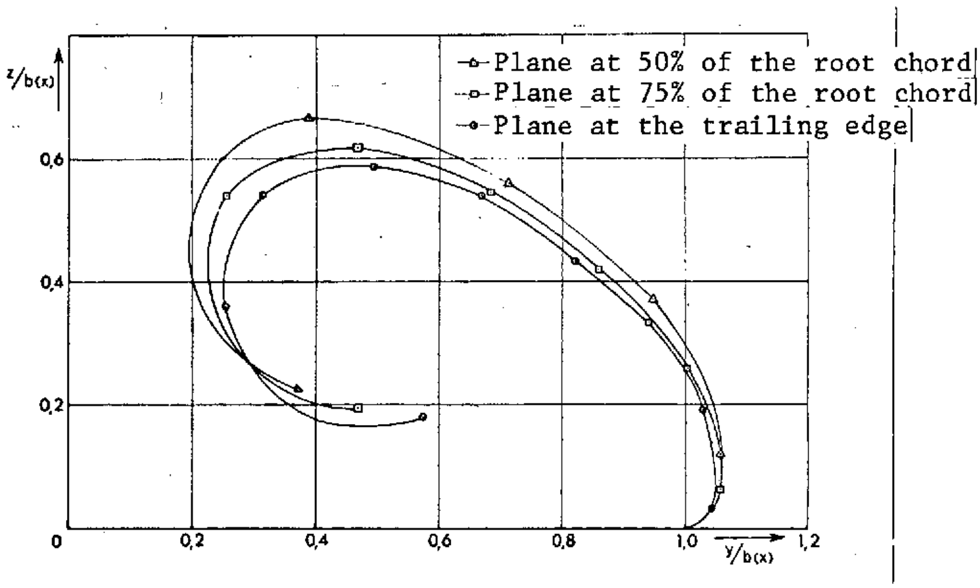

Figure 6. Verification of the taper of the flow

agreement of the results of these three methods of calculation could be that the wing examined is such that it cannot be properly analyzed by slender body theory. The deviation of the flow from conical flow, which can be seen by comparing the results of Smith and Sacks, seems to be greater than the deviation which occurs in our calculations (Figure 6). As far as the height displacement of the sheet is concerned, the effect of the deviation from conical flow is in the same direction in the two cases. Also J. K. Harvey [13] and P. T. Fink, et al. [14] found from experiments that the condition for conical flow for delta wings having semi-aperture angles at the apex of 10° is not well satisfied except in transverse planes which do not go beyond 40% of the total chord of the wing. Figure 8 shows the variation of the normal force coefficient as a function of incidence angle. It is known that the more slender a wing is, the greater the value of C_N will become for a given incidence angle. This explains the fact why the values obtained by the method of A. H. Sacks [7](wing with $\varepsilon = 14°$) and with the present method (wing with $\varepsilon = 15°$) are closer to the experimental results (wing with $\varepsilon = 14°$) of D. H. Peckham [12] than those given by J. H. B. Smith [3]. The results of this latter offer can only be applied with a high degree of accuracy for wings which are much more slender ($\varepsilon < 10°$). As a reference curve, this figure also shows the $C_N(i)$ corresponding to the theory of Jones, which is valid for a slender wing without leading edge sheets.

In Figure 9, we show the variation of the center of pressure as a function of incidence angle. We find that our numerical results agree well with the experimental values [12].

In order to gain an idea of the computation effort required to obtain the results presented, it is interesting to note that the calculation program occupies 500 K of rapid access memory.

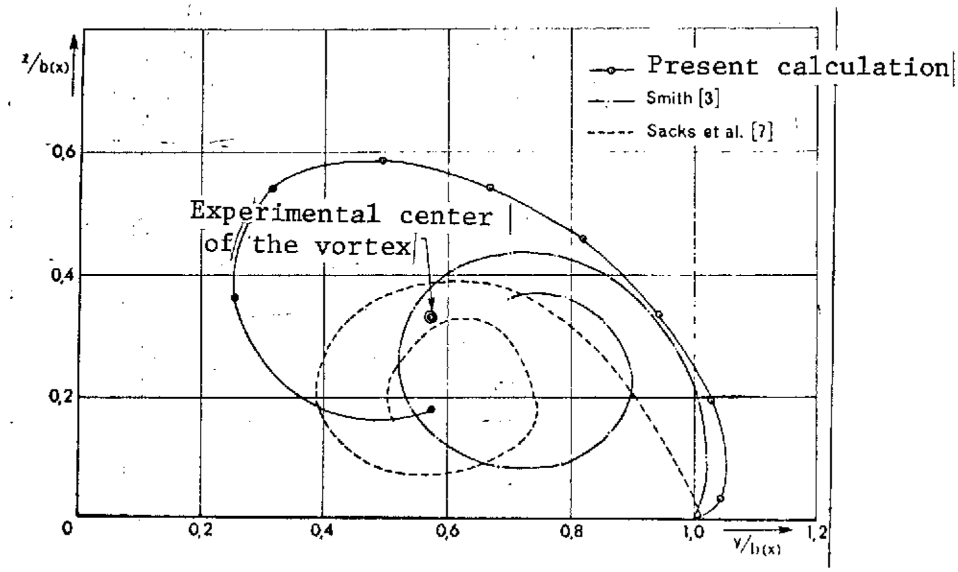

Figure 7. Trace of the sheet in the plane transverse to the trailing edge for a = 1

Each incidence angle required approximately 15 iteration cycles corresponding to 15 minutes of calculation time on a IRIS 80 computer.

V. CONCLUSION

We have presented the first results obtained during the development of a calculation method for vortex sheets which emanate from wing leading edges. The conditions assumed are such that the criteria for the application of classical methods within the framework of slender body theory or possibly conical flow are not satisfied.

It was found that if the conditions over a wing are such that from the geometric point of view, they lie outside of the scope of the simplified theories mentioned above, the method presented gives results which agree satisfactorily with experiment. There is a certain inaccuracy in the velocity field near the apex, which

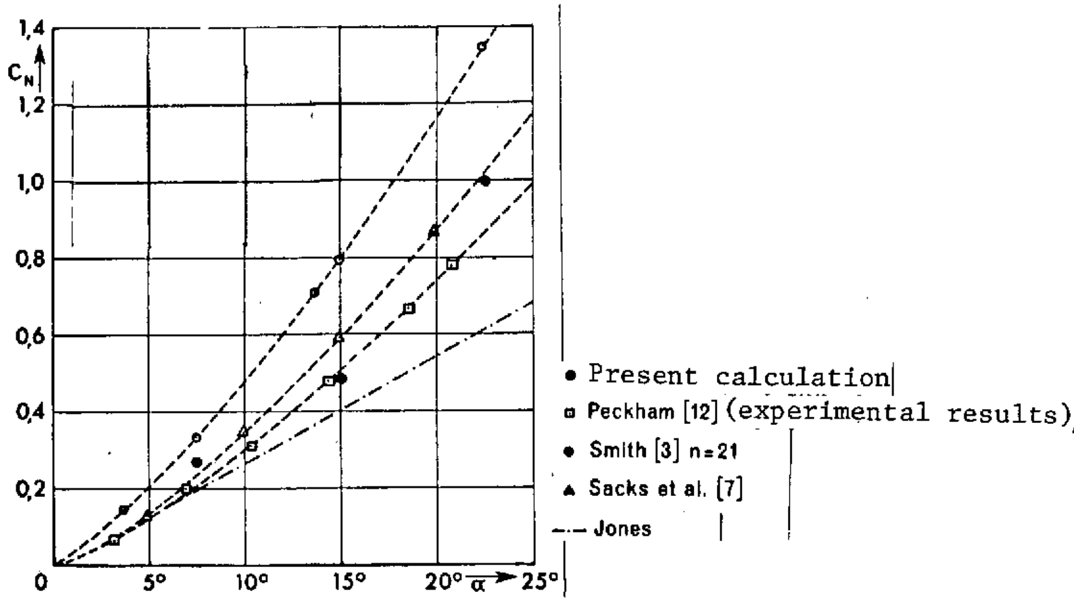

Figure 8. Normal force coefficient

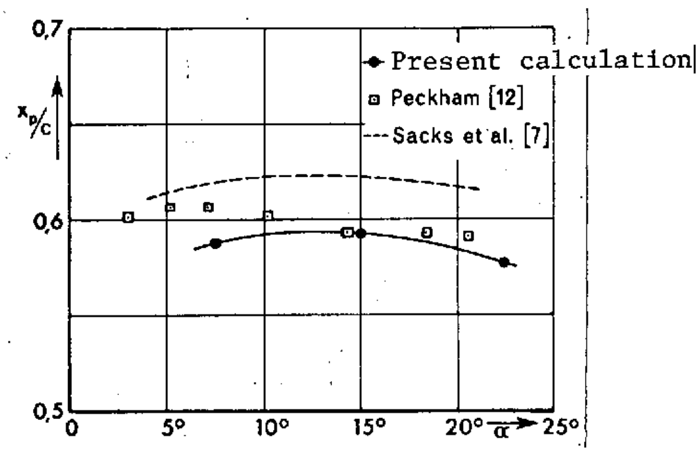

Figure 9. Evolution of the pressure center

is probably due to the fact that the calculations are performed with a very close network of discrete singularities. This inaccuracy could be removed in the future by using the conical nature of the flow within this limited region.

Without any modification, this method could be applied to the treatment of wings having a more complicated geometry, such as wings with varying leading edge or wings with a non-plane skeleton (Concorde type wings).

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